PHYS 301
Electricity and Magnetism

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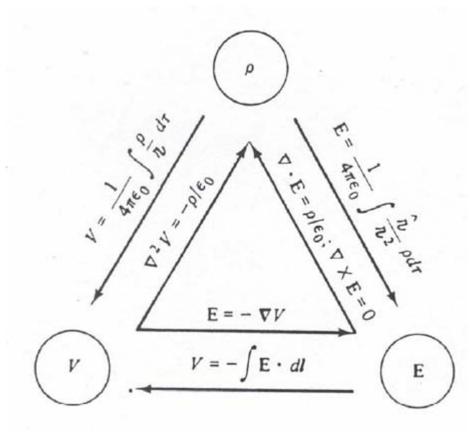
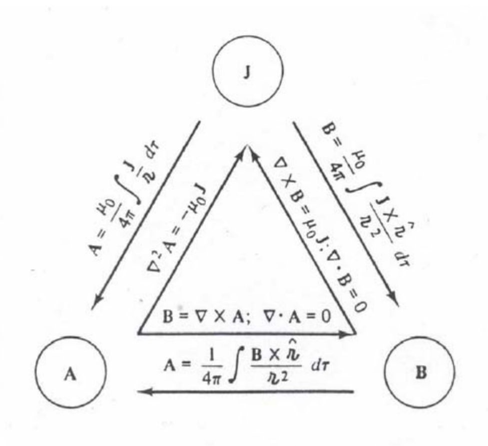
Fall 2019

Today!

- Magnetic fields
  - Law of Biot-Savart
  - Ampere’s Law

electrostatics **magnetostatics**



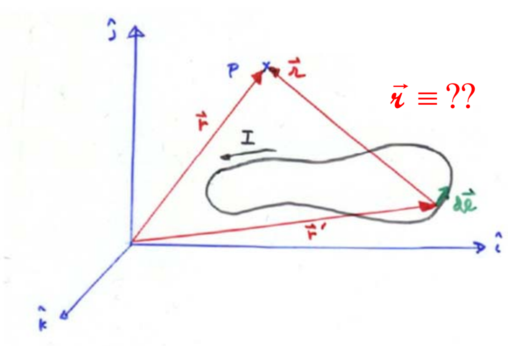


magnetostatics

The Law of Biot-Savart

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{line} \frac{\vec{I} \times \hat{\mathscr{r}}}{\mathscr{r}^2} dl' = \frac{\mu_o I}{4\pi}\int_{line} \frac{d\vec{l}' \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

for constant I



$\vec{\mathscr{r}} \equiv ??$

[for steady currents!]

$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{vol} \frac{\vec{K} \times \hat{\mathscr{r}}}{\mathscr{r}^2} dA'$$

where $\vec{K} = \vec{K}(\vec{r}')$

Or, more generally,

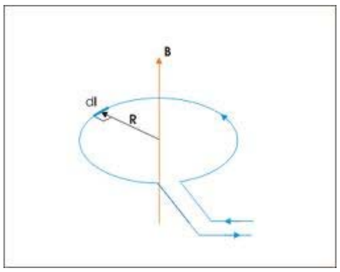
$$\vec{B}(\vec{r}) = \frac{\mu_o}{4\pi}\int_{vol} \frac{\vec{J} \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

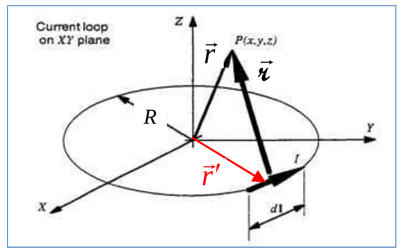
where $\vec{J} = \vec{J}(\vec{r}')$

magnetostatics

***E.g.*, the "axial" magnetic field for a circular current loop carrying a current, *I*, is**

$$\vec{B} = \frac{\mu_o I}{2}\frac{R^2}{(R^2+z^2)^{3/2}}\hat{k}$$





Trickier!!

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THE DIVERGENCE OF $\vec{B}$

$\vec{J} = \vec{J}(\vec{r}\,')$

$$\vec{\nabla}\cdot\vec{B} = \frac{\mu_o}{4\pi}\int_{vol}\vec{\nabla}\cdot\left(\frac{\vec{J}\times\hat{𝓇}}{𝓇^2}\right)d\tau'$$

$$\vec{\nabla}\cdot\vec{B} = \frac{\mu_o}{4\pi}\int\left\{\frac{\hat{𝓇}}{𝓇^2}\cdot(\vec{\nabla}\times\vec{J}) - \vec{J}\cdot\left(\vec{\nabla}\times\frac{\hat{𝓇}}{𝓇^2}\right)\right\}d\tau'$$

= 0 why?!   = 0 why?!

$$\therefore \quad \vec{\nabla}\cdot\vec{B} = 0$$

One of Maxwell's equations – Fundamental!!

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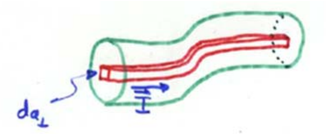
THE CONTINUITY EQUATION

$$\vec{J} \equiv \rho \vec{v} = \text{ volume current density} = \frac{d\vec{I}}{dA_\perp}$$



So we can write

$$I = \left| \vec{I} \right| = \int_{surf} \left| \vec{J} \right| dA_\perp = \int_{surf} \vec{J} \cdot d\vec{A}$$

If we integrate over a closed surface, **Gauss' theorem** allows us to write

$$\oint_{surf} \vec{J} \cdot d\vec{A} = \int_{vol} (\vec{\nabla} \cdot \vec{J})\, d\tau'$$

for the charge flowing out of the surface.

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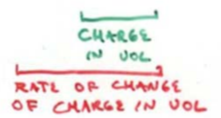
This quantity must equal the **decrease** of charge inside the volume:

$$\int_{vol} (\vec{\nabla} \cdot \vec{J})\, d\tau' = -\frac{d}{dt} \int_{vol} \rho\, d\tau' = -\int_{vol} \frac{d\rho}{dt} d\tau'$$



Since this is independent of the volume chosen,
we get the general result that

$$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$$

CONTINUITY EQUATION

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THE CURL OF $\vec{B}$

$$\vec{\nabla}\times\vec{B} = \mu_o \vec{J}(\vec{r})$$

AMPERE'S LAW – (PART OF!) ONE OF MAXWELL'S EQUATIONS

* *It's* always *true – but not always* useful*!*

magnetostatics

INTEGRAL FORM OF AMPERE'S LAW:

$$\int_{surf} (\vec{\nabla}\times\vec{B})\cdot d\vec{A}' = \oint_{line} \vec{B}\cdot d\vec{l}' = \mu_o \int_{surf} \vec{J}\cdot d\vec{A}'$$

Stokes' theorem

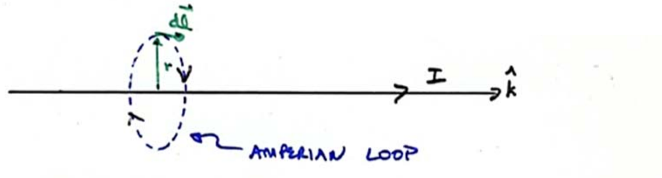
$$\oint_{line} \vec{B}\cdot d\vec{l}' = \mu_o I_{encl}$$

- *Plays role in magnetostatics parallel to that of Gauss' law in electrostatics!*

magnetostatics

EX: LONG STRAIGHT WIRE:

- EXPECT A "CIRCUMFERENTIAL" FIELD!



So,

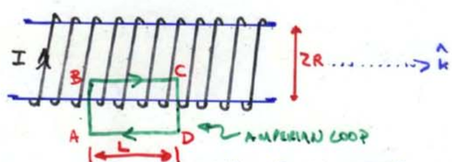
- EXPECT |B| = CONSTANT ON THE LOOP

$$\oint \vec{B}\cdot d\vec{l}' = \oint B\, dl' = B\oint dl' = B\,2\pi r = \mu_o\, I_{encl} = \mu_o\, I$$

$$\therefore\ \vec{B} = \frac{\mu_o\, I}{2\,\pi\, r}\,\hat{\phi}$$

magnetostatics

EX: A LONG SOLENOID ("IDEAL SOLENOID")



* TIGHTLY WOUND ; LENGTH >> DIAMETER.

* STRONG AXIAL $\vec{B}$ INSIDE SOLENOID
WEAK " " OUTSIDE " "

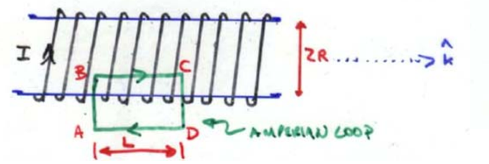
* EXPECT NO RADIAL AND NO TANGENTIAL COMPONENTS TO $\vec{B}$ – WHY?

SO, WE HAVE

$$\oint \vec{B}\cdot d\vec{l} = \underbrace{\int_a^b \vec{B}\cdot d\vec{l}}_{=0\ (\vec{B}\perp d\vec{l})} + \int_b^c \vec{B}\cdot d\vec{l} + \underbrace{\int_c^d \vec{B}\cdot d\vec{l}}_{=0\ (\vec{B}\perp d\vec{l})} + \underbrace{\int_d^a \vec{B}\cdot d\vec{l}}_{\approx 0\ (\text{OUTSIDE})}$$

magnetostatics

EX: A LONG SOLENOID ("IDEAL SOLENOID")



$$\oint \vec{B}\cdot d\vec{l} = \int_b^c B\,dl = B\int_b^c dl$$

$$\oint \vec{B}\cdot d\vec{l} = B\cdot L = \mu_0 I_{enc} = \mu_0 \cdot I \cdot n \cdot L$$

(n = # TURNS PER UNIT LENGTH)

$$\therefore \vec{B} = -\mu_0 n I \hat{k}$$

Maxwell's Equations

Where do we stand now?

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_o$	ELECTROSTATICS	Gauss' Law $(\vec{E})$
$\vec{\nabla}\times\vec{E} = 0$		(Faraday's Law)
$\vec{\nabla}\cdot\vec{B} = 0$	MAGNETOSTATICS	Gauss' Law $(\vec{B})$
$\vec{\nabla}\times\vec{B} = \mu_o\vec{J}$		Ampere's Law ↑ (-Maxwell)

with $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

We're not quite finished!

- electric fields in matter – PHYS 425
- magnetic fields in matter – PHYS 425
- time dependent fields